KENDRIYA VIDYALAYA SANGATHAN
GUWAHATI REGION

SPECIAL STUDY MODULE – MATHEMATICS

FOR

CLASS – XII (2015-16)

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TARGET – 50

MINIMUM LEARNING MATERIAL

CLASS - XII

Topics	Marks
1. Inverse trigonometric function	4
2. Properties of determinants	4
3. Solution of equations using matrix method	4
4. Continuity	4
5. Logarithmic differentiation	4
6. Word problems maxima/minima	6
7. Integral	4
8. Vector	4
9. . Three dimensional geometry	4
10. Linear programming	6
11. Probability (Baye's Theorem)	6
Total Marks	50

MINIMUM LEARNING MATERIAL

TARGET – 50

1. Inverse trigonometric function (4 marks)

Use $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$ $\tan^{-1}x-\tan^{-1}y=\tan^{-1}\left(\frac{x-y}{1+xy}\right)$ $2\tan^{-1}x=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$

1. Prove that: $\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{1}{8}\right)=\frac{\pi}{4}$
2. Prove that: $\tan^{-1}\left(\frac{1}{7}\right)+\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left[\frac{1}{3}\right]+\tan^{-1}\left(\frac{1}{8}\right)=\frac{\pi}{4}$
3. Prove that: $2\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{7}\right)=\tan^{-1}\left(\frac{31}{17}\right)$
4. Find the value of $\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\left(\frac{x-y}{x+y}\right)$
5. $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x, \frac{-1}{\sqrt{2}}\le x\le 1$
6. Prove the following: $\tan^{-1}x+\tan^{-1}\left(\frac{2x}{1-x^2}\right)=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$.
7. Prove that: $\cos^{-1}\left(\frac{4}{5}\right)+\cos^{-1}\left(\frac{12}{13}\right)=\cos^{-1}\left(\frac{33}{65}\right)$
8. Prove that: $\sin^{-1}\left(\frac{8}{17}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\cos^{-1}\left(\frac{36}{85}\right)$.
9. Solve $\tan^{-1}2x+\tan^{-1}3x=\frac{\pi}{4}$
10. Find the value of x if $\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$

11. $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2}, x\in(0,\frac{\pi}{4})$

12. Solve $\tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}\tan^{-1}x,(x>0)$

13. Express in the simplest form $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right), x\neq 0$

14. Solve 2 $\tan^{-1}$(cos x) = $\tan^{-1}$(2 cosec x) .

15. Prove $\tan^{-1}\sqrt{x}=\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right), x\in(0,1)$

2. Properties of determinants(4 marks)

Using the properties of determinants, show that

1. $\begin{vmatrix}1 & x & x^2\\ 1 & y & y^2\\ 1 & z & z^2\end{vmatrix}$ = (x – y)(y –z)(z – x) (C.B.S.E. 1991)

2. $\begin{vmatrix}1 & 1 & 1\\ \alpha & \beta & \gamma\\ \beta\gamma & \gamma\alpha & \alpha\beta\end{vmatrix}=(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)$ C.B.S.E. 2006, 04)

3. $\begin{vmatrix}1 & a & a^3\\ 1 & b & b^3\\ 1 & c & c^3\end{vmatrix}$ = (a - b)(b - c)(c - a)(a + b + c) (C.B.S.E. 1997, 96 ,2000, 2003C)

4. $\begin{vmatrix}x & y & z\\ x^2 & y^2 & z^2\\ x^3 & y^3 & z^3\end{vmatrix}$ = xyz(x – y)(y – z)(z – x) (C.B.S.E. 2000)

5. $\begin{vmatrix}\alpha & \beta & \gamma\\ \alpha^2 & \beta^2 & \gamma^2\\ \beta+\gamma & \gamma+\alpha & \alpha+\beta\end{vmatrix}=(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)(\alpha+\beta+\gamma))$ (C.B.S.E. 2008, 05)

6. $\begin{vmatrix}x+4 & 2x & 2x\\ 2x & x+4 & 2x\\ 2x & 2x & x+4\end{vmatrix}$ = (5x + 4)(x – 4)^2 (C.B.S.E. 1996)

7. $\begin{vmatrix}a & a+b & a+2b\\ a+2b & a & a+b\\ a+b & a+2b & a.\end{vmatrix}$ = 9 (a + b) b^2 (C.B.S.E. 2008, 02)

8. $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a + b + c)^3$ (C.B.S.E. 2007, 06, 04,2000C,1998,97)

9. $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a + b + c)^3$ (C.B.S.E. 2006, 04, 1999)

10. $\begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix} = a^2 (a + x + y + z)$ (C.B.S.E. 2003)

11. Show that

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$

12. Using the Properties of Determinants Prove that $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1 + a^2+b^2+c^2$

13. Using the properties of Determinants,show that $\begin{vmatrix} (b+c)^2 & ab & ac \\ ab & (a+c)^2 & bc \\ ca & cb & (a+b)^2 \end{vmatrix} = 2abc(a+ b + c)^3$

14. the Show that x = 2 is a root of equation $\begin{vmatrix} x & -6 & 1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$ and solve it Completely.

15.) Using properties of determinants, solve the following for x:

$$\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$$

3. Matrix (4 marks)

1. Using matrices, solve the following system of equation:
   $2x-3y+5z=11,\ 3x+2y-4z=-5,\ x+y-2z=-3$

2. Using matrix method, solve the following system of equations :

   3x-2y+3z=8. , 2x+y- z=1, 4x-3y+2z=4.

3. An amount of Rs 600 crores is spent by the government in three schemes. Scheme A is for saving girl child from the cruel parents who don't want girl child and get the abortion before her birth. Scheme B is for saving of newlywed girls from death due to dowry. Scheme C is planning for good health for senior citizen. Now twice the amount spent on Scheme C together with amount spent on Scheme A is Rs 700

crores. And three times the amount spent on Scheme A together with amount spent on Scheme B and Scheme C is Rs 1200 crores. Find the amount spent on each Scheme using matrices? **What is the importance of saving girl child from the cruel parents who don't want girl child and get the abortion before her birth?**

4. A school has to reward the students participating in co-curricular activities (Category I) ,with 100% attendance (Category II) and class toppers (Category III) in a function. The sum of the numbers of all the three category students is 6. If we multiply the number of students of category III by 2 and added to the number of students of category I , we get 7. By adding the number of students of category II and III with three times the number of students of category I we get 12.Form the matrix equation and solve it. ***Do you think the school should add one more category to motivate the students for cleanliness? Give your idea in brief.***

5. if $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$; Then show that A^3-23A-40$\boldsymbol{I}$=0

6. If
$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix};$$ verify that $A^2 - 4A - 5I = 0$

7. $If, A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} and, I = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ Find 'k' so that A^2=kA-2I. Ans : k=1

8. By using elementary operations find the inverse of the matrix $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

9. There are three families. First family consists of 2 male members, 4 female members and 3 children. Second family consists of 3 male members, 3 female members and 2 children. Third family consists of 2 male members, 2 female members and 5 children. Male member earns Rs 500 per day and spends Rs 300 per day. Female member earns Rs 400 per day and spends Rs 250 per day child member spends Rs 40 per day. Find the money each family saves per day using matrices? What is the necessity of saving in the family?

4. Continuity (4 marks)

A function f is said to be continuous at x =a if LHL = RHL = f(a)

Find the unknown constant if the functions are continuous

(1) $f(x) = \begin{cases} k\,x^2, & x \le 2 \\ 3, & x > 2 \end{cases}$

2. $f(x)=\begin{cases} 2x-1, & if \quad x<2 \\ a, & if \quad x=2 \\ x+1, & if \quad x>2 \end{cases}$

(3) $f(x)=\begin{cases} 2x+1, & if \quad x<2 \\ k, & if \quad x=2 \\ 3x-1, & if \quad x>2 \end{cases}$

(4) $f(x)=\begin{cases} 3ax+b, if & x>1 \\ 11, if & x=1 \\ 5ax-2b, if & x<1 \end{cases}$

(5) $f(x)=\begin{cases} 5, & x\le 2 \\ ax+b, & 2<x<10 \\ 21, & x\ge 10 \end{cases}$

(6) $f(x)=\begin{cases} kx+1, & x\le \pi \\ \cos x, & if \quad x>\pi \end{cases}$

(7) $f(x)=\begin{cases} k(x^2+2), & if \quad x\le 0 \\ 3x+1, & if \quad x>0 \end{cases}$

(8) $f(x)=\begin{cases} \lambda(x^2-2x), & if \quad x\le 0 \\ 4x+1, & if \quad x>0 \end{cases}$

9. Find the relation between a and b so that $f(x)=\begin{cases} ax+1, & x\le 3 \\ bx+3, & x>3 \end{cases}$ is continues at x = 3.

10. The function $f(x)$ is defined as follows: $f(x)=\begin{cases} x^2+ax+b, & 0\le x<2 \\ 3x+2, & 2\le x\le 4 \\ 2ax+5b, & 4<x\le 8 \end{cases}$

If f is continuous on [0,8] find the values of a and b.

5. Logarithmic differentiation(4 marks)

Differentiate w.r. to x

(1) If y =xcos x + (cos x)x, find $\frac{dy}{dx}$.

(2) If y= $x^x+(\cos x)^{\sin x}$ find $\frac{dy}{dx}$

(3) If x=a sin pt and y=b cospt, find the value of $\frac{d^2y}{dx^2}$ at t=0?

(4) If x= a(cost+log tant/2) and y= asint find $\frac{dy}{dx}$.

(5) If x = X = 4 $\sin^3$t & y = 4 $\cos^3$t, find $\frac{d^2y}{dx^2}$ $at\, t = \frac{\pi}{3}$

(6) Differentiate $\sqrt{\frac{(x-3)(x^2+4)}{3x^2+4x+5}}$ with respect to x.

(7) ify= $(\sin x)^x + \tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$ find $\frac{dy}{dx}$.

(8) If $x^y = e^{x-y}$, then prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$

(9) $\frac{dy}{dx}$ If $x^y + y^x = a^b$

(10) If (cos x) ^{y} = (cos y)^x find $\frac{dy}{dx}$

7. Word problems maxima/minima (6 marks)

1. A given quantity of metal is to be cast into a half cylinder with a rectangular base and semicircular ends. Show that in order that the total surface area is to be minimum, the ratio of the length of the cylinder to the diameter of the its semicircular ends is π : (π +2).

2. A window is in the form of a rectangle above which there is a semi-circle. If the perimeter of the window is p cm, show that the window will allow the maximum possible light only when the radius of the semi-circle is $\frac{p}{\pi+4}$ cm.
3. A window is in the form of a rectangle surmounted by a semi-circular opening. The total perimeter of the window is 10 m. find the dimensions of the window so as to admit maximum light through the whole opening.
4. An open box with a square base is to be made out of a given quantity of sheet of area a^2. Show that the maximum volume of the box is $\frac{a^3}{6\sqrt{3}}$.
5. If length of three sides of a trapezium other than base are equal to 10cm, then find the area of the trapezium when it is maximum.
6. Show that the semi vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1}\frac{1}{3}$.

   (For self practice)

7. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.
8. An open topped box is to be constructed by removing equal squares from each corner of a 3 metre by 8 metre rectangular sheet of aluminium and folding up the sides. Find the volume of the largest such box.

9. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $2R/\sqrt{3}$.
10. Show that the volume of the largest cone that can be inscribed in a sphere is 8/27 of the volume of the sphere.
11. Show that of all the rectangles of given area, the square has the smallest perimeter.
12. If the sum of the lengths of the hypotenuse and a side of a right angled triangle is given, show that the area of the triangle is maximum when the angle between them is $\pi/3$.

Integral (4 marks)

1. Evaluate $\int \frac{1}{\cos(x-a)\cos(x-b)}dx$
2. Evaluate $\int e^{x}\frac{x^{2}+1}{(x+1)^{2}}dx$
3. Evaluate $\int \frac{x+1}{(x+3)^{3}}e^{x}\,dx$
4. Evaluate $\int \frac{\cos x dx}{(1+\sin x)(2+\sin x)}$
5. Evaluate $\int e^{x}\left(\frac{1-\sin x}{1-\cos x}\right) dx$.
6. Evaluate $\int \frac{dx}{\sin x+\sqrt{3}\cos x}$
7. $\int \frac{\cos x+\sin x}{9+16\sin 2x}dx$
8. Evaluate: $\int \frac{dx}{x(x^{n}+1)}$
9. Evaluate the following: $\int_{1}^{4}\left[|x-1|+|x-2|+|x-3|\right]dx$
10. Evaluate: $\int_{0}^{\pi}\frac{x\tan x}{\sec x+\tan x}dx$

Three dimensional geometry

1. The eq. of the lines are $\frac{x-1}{1}=\frac{y-2}{-3}=\frac{z-3}{2}$ and $\frac{x-4}{2}=\frac{y-5}{3}=\frac{z-6}{1}$. Find the shortest distance between the above lines.

2. Find the shortest distance between the lines

   $\vec{r}=\hat{i}+\hat{j}+\lambda(2\hat{i}-\hat{j}+\hat{k})$ and

   $\vec{r}=2\hat{i}+\hat{j}-\hat{k}+\mu(3\hat{i}-5\hat{j}+2\hat{k})$

3. Find the equation of the plane determined by the points A(3,-1,2), B(5,2,4) and C (-1,-1,6) .Also find the distance of the point P(6,5,9) from the plane .

4. Find the image of the point (1,3,4) in the plane x-y+z =5
5. Find the image of the point (1, 2, -1) in the plane 2x+y-z=2
6. By computing the shortest distance between the following lines, determine whether they intersect or not?
   $\vec{r}=\hat{i}-\hat{j}+\lambda(2\hat{i}+\hat{k})$; $\vec{r}=2\hat{i}-\hat{j}+\mu(\hat{i}-\hat{j}-\hat{k})$

7. Find the equation of the plane which contains line of intersection of planes $\vec{r}.(\hat{i}+2\hat{j}+3k)-4=0$, $\vec{r}.(2\hat{i}+\hat{j}-k)+5=0$ and which passes through the Point (1,0,-2).

8. Find the equation of the plane through $2\hat{i}+\hat{j}-\hat{k}$ and passing through the line of intersection of the planes $\vec{r}\cdot(\hat{i}+3\hat{j}-\hat{k})=0$and $\vec{r}\cdot(\hat{j}+2\hat{k})=0$.

9. Find the angle between the line $\frac{x+1}{2}=\frac{3y+5}{9}=\frac{3-z}{-6}$ and the plane $10x+2y-11z=3$.

10. Find the equation of planes passing through the point (-1,-1,2) and perpendicular to the planes x + 2y - 3z =1 and 5x – 4y + 3z = 5

Vectors

1. Let $\vec{a}=\hat{\imath}+4\hat{\jmath}+2\hat{k}$, $\vec{b}=3\hat{\imath}-2\hat{\jmath}+7\hat{k}$ and $\vec{c}=2\hat{\imath}-\hat{\jmath}+4\hat{k}$, find a vector $\vec{d}$ which is perpendicular to both vectors $\vec{a}$ *and* $\vec{b}$ and $\vec{c}.\vec{d}=15$.
2. If a and b are unit vectors and θ is the angle between them, then prove that cos θ/2 = ½ |a + b|

3. . If $\vec{a},\vec{b},\vec{c}$ be three mutually perpendicular vectors of equal magnitude ,then show that $(\vec{a}+\vec{b}+\vec{c})$ is equally inclined to the vectors $\vec{a},\vec{b}$ and $\vec{c}$.

4. If the sum of two unit vector $\hat{a}$ *and* $\hat{b}$ is a unit vector, find the magnitude of their difference.
5. Prove that $[\vec{a}+\vec{b} \quad \vec{b}+\vec{c} \quad \vec{c}+\vec{a}] = 2[\vec{a}, \vec{b}, \vec{c}]$
6. If vectors a, b, c satisfy the condition $\left(\vec{a}+\vec{b}+\vec{c}\right)$=0and $|\vec{a}| = 1$, $|\vec{b}| = 4$, $|\vec{c}| = 2$ than find the value of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$
7. If $\vec{\alpha} = 3\hat{i} - \hat{j}$ *and* $\vec{\beta} = 2\hat{i} + \hat{j} - 3\hat{k}$. Express $\vec{\beta}$ *as a sum of two vectors* $\vec{\beta}_1$ *and* $\vec{\beta}_2$, where $\vec{\beta}_1$ is Parallel to $\vec{\alpha}$ *and* $\vec{\beta}_2$ *is perpendicular to* $\vec{\alpha}$.

Linear programming problems (6 marks)

1. Niharika wants to invest at most Rs.12000 in Saving certificate (SC) and National saving bonds (NSB).She has to invest at least Rs.2000 in SC and at least Rs.4000 in NSB. If the rate of interest on SC is 8% pa. And the rate of interest on NSB 10% pa. How much money should she invest to earn maximum yearly income? Also find the maximum income.
2. A dealer wishes to purchase number of fans and sewing machines . He has only *Rs.* 5760 to invest and has a space for at most 20 items . A fan cost him Rs. 360 and sewing machine Rs. 240 . His expectation is that .He can sell a fan at profit of Rs. 22and sewing machine at a profit of Rs. 18 . Assuming that he can sell all the items that he can buy , how should invest his money in order to maximize the profit ?Formulate this as a linear programming problem and solve it graphically .
3. A diet is to contain at least 80 units of vitamin A and 100 units of minerals. Two foods $F_1$and F_2 are available. Food F_1 costs Rs 4 per unit food and F_2 costs Rs 6 per unit. One unit of food F_1 contains 3 units of vitamin A and 4 units of minerals. One unit of food F_2 contains 6 units of vitamin A and 3 units of minerals. Formulate this as a linear programming problem. Find the minimum cost for diet that consists of mixture of these two foods and also meets the minimal nutritional requirements?
4. A dietician wishes to mix two types of foods in such a way that vitamin contents of the mixture contain at least 8 units of vitamin A and 10 units of vitamin C. Food one ,contains 2 units/Kg of vitamin A and 1 unit/Kg of vitamin C .Food second contains 1 unit/Kg of vitamin A and 2 units/Kg of vitamin C. It costs Rs.50 per kg to food one and Rs70 perKg to purchase food second. Formulate this problem as a linear programming problem to minimise the cost of such a mixture.
5. An aero plane can carry a maximum of 200 passengers. A profit of Rs.400 is made on each first class ticket and a profit of Rs. 300 is made on each second class ticket. The airline reserves at least 20 seats for first class. However at least four times as many passengers prefer to travel by second class than by first class. Determine how many tickets of each type must be sold to maximize profit for the airline. Form an L.P.P. and solve it graphically. Do you think that in India people would like to travel by economy class than executive class? Give reason.

Probability

1. In a bolt factory, three machines A, B, C manufacture 25%, 35% and 40% of the total production respectively. Of their respective output, 5%, 4% and 2% are defective. A bolt is drawn at random from the

total product and it is found to be defective. Find the probability that it was manufactured by the machine C.

2. By examining the chest X-ray , the probability that TB is detected when a person actually suffering is 0.99 . The probability of incorrect diagnosis is 0.001. In a certain city one in thousand persons suffer from TB . A person selected at random and is diagnosed to have TB. What is the chance that he actually has TB.

3. There are three coins.One is a two headed coin (having head on both faces) ; another a biased coin that comes up with tail 25% of times. And third is an unbiased coin . One of the three coins is chosen at random and tossed. If it shows head ,what is the probability that it was a two headed coin?

4. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let ¾ be the probability that he knows the answer and ¼ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with the probability 1/4. What is the probability that the student knows the answer given that he answered it correctly?
5. A card from a pack of 52 cards is lost , From the remaining cards of the pack, two card are drawn and are found to be both diamonds. Find the probability of lost card being a diamond.

6. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident involving a scooter ,a car and a truck are 0.01, 0.03 and 0.15 respectively. One of the insured person meets with an accident. What is the probability that he is a scooter driver . Which mode of transport would you suggest to a student and why?

7. Suppose that the reliability of a HIV test is specified as follows: Of people having HIV, 90% of the test detect the disease but 10% go undetected. Of people free of HIV, 99% of the test are judged HIV–ive but 1% are diagnosed as showing HIV+ive. From a large population of which only 0.1% have HIV, one person is selected at random, given the HIV test, and the pathologist reports him/her as HIV+ive. What is the probability that the person actually has HIV? What moral advice should you give to your peers so that they will not get infected by HIV virus?

INVERSE TRIGONOMETRIC FUNCTION

Ans. 1

$$\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{8}\right) = \tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{5}}{1-\frac{1}{2}\frac{1}{5}}\right) + \tan^{-1}\frac{1}{8}$$

$$= \tan^{-1}\left(\frac{\frac{7}{10}}{\frac{9}{10}}\right) + \tan^{-1}\frac{1}{8} \qquad = \tan^{-1}\left(\frac{7}{9}\right) + \tan^{-1}\frac{1}{8} = \tan^{-1}1 = \frac{\pi}{4}$$

Ans. 2

$$\text{L.H.S.} = \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{8}$$

$$= \tan^{-1}\left(\frac{\frac{1}{5}+\frac{1}{7}}{1-\frac{1}{5}\times\frac{1}{7}}\right) + \tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{8}}{1-\frac{1}{3}\times\frac{1}{8}}\right) \qquad \left[\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}\right]$$

$$= \tan^{-1}\left(\frac{7+5}{35-1}\right) + \tan^{-1}\left(\frac{8+3}{24-1}\right)$$

$$= \tan^{-1}\frac{12}{34} + \tan^{-1}\frac{11}{23}$$

$$= \tan^{-1}\frac{6}{17} + \tan^{-1}\frac{11}{23}$$

$$= \tan^{-1}\left(\frac{\frac{6}{17}+\frac{11}{23}}{1-\frac{6}{17}\times\frac{11}{23}}\right)$$

$$= \tan^{-1}\left(\frac{138+187}{391-66}\right)$$

$$= \tan^{-1}\left(\frac{325}{325}\right) = \tan^{-1}1$$

$$= \frac{\pi}{4} = \text{R.H.S.}$$

Ans. 3

$$2\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1}\left(\frac{31}{17}\right)$$

Ans. 4

$$\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x-y}{x+y}\right) = \tan^{-1}\frac{x(x+y)-y(x-y)}{y(x+y)+x(x-y)}$$

$$= \tan^{-1}\frac{x^2+xy-yx+y^{\ 2}}{yx+y^2+x^2-xy)} \quad \tan^{-1}1 = \frac{\pi}{4}$$

Ans. 5

Put x = cos2θ $\Rightarrow$ $\theta = \frac{1}{2}cos^{-1}$x

Thus,LHS = $\tan^{-1}\left[\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\right]$

$\Rightarrow\ = \tan^{-1}\left[\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}\right]$

$\Rightarrow\ = \tan^{-1}\left[\frac{1-\tan\theta}{1+\tan\theta}\right] \qquad = \pi/4 - \theta \qquad = \pi/4 - \frac{1}{2}\cos^{-1}x \qquad$ =RHS

Ans. 6

LHS = $\tan^{-1} x + \tan^{-1} 2x = 3\tan^{-1} 3x = \tan^{-1}\left(\frac{3x - x^3}{1-3x^2}\right)$

Ans. 7

$\cos^{-1}\left(\frac{4}{5}\right) = \tan^{-1}\frac{3}{4}, \cos^{-1}\left(\frac{12}{13}\right) = \tan^{-1}\left(\frac{5}{12}\right)$

LHS = $\cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \tan^{-1}\frac{3}{4} + \tan^{-1}\left(\frac{5}{12}\right)$

$= \tan^{-1}\left(\frac{56}{33}\right) = \cos^{-1}\frac{33}{65}$

Ans 8

Proceed as above

Ans 9

LHS = $\tan^{-1}\frac{5x}{1-6x^2} = \frac{\pi}{4}$

$\Rightarrow \frac{5x}{1-6x^2} = 1$

$\Rightarrow 6x^2 + 5x - 1 = 0$ solving we get

Ans. 10

$$\tan^{-1}\frac{x-1}{x-2} + \tan^{-1}\frac{x+1}{x+2} = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left[\frac{\frac{x-1}{x-2} + \frac{x+1}{x+2}}{1-\left(\frac{x-1}{x-2}\right)\left(\frac{x+1}{x+2}\right)}\right] = \frac{\pi}{4} \qquad \left[\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}\right]$$

$$\Rightarrow \tan^{-1}\left[\frac{(x-1)(x+2)+(x+1)(x-2)}{(x+2)(x-2)-(x-1)(x+1)}\right] = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left[\frac{x^2+x-2+x^2-x-2}{x^2-4-x^2+1}\right] = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left[\frac{2x^2-4}{-3}\right] = \frac{\pi}{4}$$

$$\Rightarrow \tan\left[\tan^{-1}\frac{4-2x^2}{3}\right] = \tan\frac{\pi}{4}$$

$$\Rightarrow \frac{4-2x^2}{3} = 1$$

$$\Rightarrow 4 - 2x^2 = 3$$

$$\Rightarrow 2x^2 = 4 - 3 = 1$$

$$\Rightarrow x = \pm\frac{1}{\sqrt{2}}$$

Ans. 11

LHS = $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right) = \cot^{-1}\left(\frac{\sqrt{(\cos\frac{x}{2}+\sin\frac{x}{2})^2}+\sqrt{(\cos\frac{x}{2}-\sin\frac{x}{2})^2}}{\sqrt{(\cos\frac{x}{2}+\sin\frac{x}{2})^2}-\sqrt{(\cos\frac{x}{2}-\sin\frac{x}{2})^2}}\right)$

$= \cot^{-1}\left(\cot\frac{x}{2}\right) = \frac{x}{2}$

Ans. 12

$\tan^{-1}\frac{1-x}{1+x} = \frac{1}{2}\tan^{-1}x$

$\Rightarrow \tan^{-1}1 - \tan^{-1}x = \frac{1}{2}\tan^{-1}x \qquad \left[\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}\right]$

$\Rightarrow \frac{\pi}{4} = \frac{3}{2}\tan^{-1}x$

$\Rightarrow \tan^{-1}x = \frac{\pi}{6}$

$\Rightarrow x = \tan\frac{\pi}{6}$

$\therefore x = \frac{1}{\sqrt{3}}$

Ans. 14

$\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$

Put $x = \tan\theta \Rightarrow \theta = \tan^{-1}x$

$\therefore \tan^{-1}\frac{\sqrt{1+x^2}-1}{x} = \tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right)$

$= \tan^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right) = \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$

$= \tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)$

$= \tan^{-1}\left(\tan\frac{\theta}{2}\right) = \frac{\theta}{2} = \frac{1}{2}\tan^{-1}x$

Ans. 14

2 $\tan^{-1}(\cos x) = \tan^{-1}(2 \operatorname{cosec} x)$

$\Rightarrow \tan^{-1}(2\cos x/1-\cos^2 x) = \tan^{-1}(2 \operatorname{cosec} x)$

$\Rightarrow$ 2cos x= 2 cosec x (1-cos^2 x)

$\Rightarrow$ 2 sin x cos x= 2- 2 cos ^{2} x

Ans. 15

Let $x = \tan^2\theta$. Then, $\sqrt{x} = \tan\theta \Rightarrow \theta = \tan^{-1}\sqrt{x}$.

$\therefore \frac{1-x}{1+x} = \frac{1-\tan^2\theta}{1+\tan^2\theta} = \cos 2\theta$

Now, we have:

R.H.S. $= \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\cos^{-1}(\cos 2\theta) = \frac{1}{2}\times 2\theta = \theta = \tan^{-1}\sqrt{x} =$ L.H.S.

Determinant

Ans . 1

Applying $R_2 \rightarrow R_2 - R_1$,and $R_3 \rightarrow R_3 - R_1$

Taking (y-x) and (z-x) common and expanding

Ans . 3

Applying $R_2 \rightarrow R_2 - R_1$,and $R_3 \rightarrow R_3 - R_1$

Taking (b-a) and (c-a) common and expanding

Ans. 4

Taking x,y,z common from R_1 , R_2 , R_3

applying $C_2 \rightarrow C_2 - C_1$,and $C_3 \rightarrow C_1$ and taking (y-x) and (z-x) common then expanding

Ans. 5

$R_3 \rightarrow R_3 + R_1$ and taking common (α+β+ϒ)

Applying $C_2 \rightarrow C_2 - C_1$,and $C_3 \rightarrow C_1$ and taking (β-α) and (ϒ-α) common and expanding

Ans. 6

$C_1 \rightarrow C_1 + C_2 + C_3$ After taking common (5x + 4) from first column then apply $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$ and then expanding along first column

Ans7

Applying $R_1 \rightarrow R_1 + R_2 + R_3$ taking 3(a + b) common and expanding

Ans 8

Apply $C_1 \to C_1 + C_2 + C_3$ After taking common 2(a+b+c) from first column then apply $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$ and then expanding along first column

Ans . 9

$C_1 \to C_1 + C_2 + C_3$ After taking common (a+b+c) from first column then apply $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$ and then expanding along first column

Ans. 10

Apply $C_1 \to C_1 + C_2 + C_3$ After taking common (a+x+y+z) from first column then apply $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$ and then expanding along first column

Ans. 11

$$\Delta = abc\begin{vmatrix} \frac{1}{a}+1 & \frac{1}{b} & \frac{1}{c} \\ \frac{1}{a} & \frac{1}{b}+1 & \frac{1}{c} \\ \frac{1}{a} & \frac{1}{b} & \frac{1}{c}+1 \end{vmatrix}$$

Apply $C_1 \to C_1 + C_2 + C_3$ taking $\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ common from C_1 then expanding

Ans.12

By performing $R_1 \to aR_1$, $R_2 \to bR_2$ and $R_3 \to cR_3$ and dividing the Determinant by abc , By taking common a , b, c from C_1 , C_2 and C_3 respectively.

Applying $R_1 \to R_1 + R_2 + R_3$ and $1+a^2+b^2+c^2$ common and expanding

Ans . 13

i) By performing $R_1 \to aR_1$, $R_2 \to bR_2$ and $R_3 \to cR_3$ and dividing the

Determinant by abc

ii) By taking common a , b and c from C_1 , C_2 and C_3 respectively

iii) Apply $C_1 \to C_1 - C_2$ and $C_2 \to C_2 - C_3$ and take common a+b+c from C1 and C2

iv) Apply $R_3 \to R_3 - (R_1 + R_2)$ and Apply $C_1 \to C_1 + C_2$

v) Apply $C_1 \to aC_1$, $C_2 \to bC_2$ anddividing the Det by abc

vi) Apply $C_1 \to C_1 + C_3$ and $C_2 \to C_2 + C_3$

Ans . 14

First apply $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$ and take common 5 and (x – 2) from R_2 and R_1

Then Apply $C_1 \to C_1 + C_3$Ans : x = 2 , 1 and -3.

Ans. 15

1) Apply$C_1 \to C_1 + C_2 + C_3$ and take common (3a – x) from C_1 then Apply $R_2 \to R_2 - R_1$ and$R_3 \to R_3 - R_1$ Ans : x = 0 , 0 and 3a.

MATRIX

ANS. 1

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix} and, B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$|A| = 2(-4+4) + (-6+4) + 5(3-2) = -6+5 = -1 \neq 0$$

$$\therefore A^{-1} exists$$

$$A_{11} = 0, A_{12} = 2, A_{13} = 1, A_{21} = -1, A_{22} = -9, A_{23} = -5$$
$$A_{31} = 2, A_{32} = 23, A_{33} = 13$$

$$\text{AdjA=} \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$A^{-1} = \frac{AdjA}{|A|} = (-1)\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

Now $X=A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0+6-5 \\ -22-45+69 \\ -11-25+39 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$x = 1, y = 2, z = 3$$

Ans. 2

The given equation can be written in matrix form as

$$\begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$$

or AX=B (say) ……….. (1)

now $|A|$=3(2-3)+2(4+4)+3(-6-4)

=-17 $\neq$ 0

$\therefore$ A^{-1} exists.

Now,A_{11}=+(2-3)=-1, A_{12}=-(4+4)=-8, A_{13}=+(-6-4)=-10

A_{21}=-(-4+9)=-5, A_{22}=+(6-12)=-6, A_{23}=-(-9+8)=1

A_{31}=+(2-3)=-1, A_{32}=-(-3-6)=9, A_{33}=+(3+4)=7A

$$\therefore \text{ Adj A} = \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix} \qquad \therefore A^{-1} = \frac{1}{|A|}(\text{Adj.A})$$

$$= \frac{1}{-17}\begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$$

By (1), X=A^{-1}B

$$= \frac{1}{-17}\begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}\begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$$

$$= \frac{-1}{17}\begin{bmatrix} -8-5-4 \\ -64-6+36 \\ -80+1+28 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{-1}{17}\begin{bmatrix} -17 \\ -34 \\ -51 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$\therefore$ x=1, y=2 and z=3 (ans.)

Ans. 3

For finding equations x+y+z = 600,x+2z=700,3x+y+z=1200
For finding A X and B
For finding A^{-1} and x,y,z
For value

Ans.4

For finding equations x+y+z = 6,x+2z=7,3x+y+z=12 finding A X and B

For finding A^{-1} and x,y,z

For value

Ans. 5

$$A^2 = \begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix};..A^3 = \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix}$$

Ans. 6

K = 1

Ans. 8

write A =IA , Apply R1<→ R2;R3→ R3-3R1;R1→ R1-2R2.;R3→ R3 +5R2.;R3→ 1/2R3; R1→ R1+R3.; R2→ R2-R3.;

$$A^{-1} = \begin{bmatrix} 1/2 & -1/2 & 1/2 \\ -4 & 3 & -1 \\ 5/2 & -3/2 & 1/2 \end{bmatrix}$$

Ans. 9

$$A = \begin{bmatrix} 2 & 4 & 3 \\ 3 & 3 & 2 \\ 2 & 2 & 5 \end{bmatrix}, B = \begin{bmatrix} 200 \\ 150 \\ -40 \end{bmatrix} \text{ find AB}$$

Continuity

Ans. 1

LHL = 5K

RHL = 3 and f(2) =5k $\Rightarrow$ K =3/5

Ans. 2

LHL =3

RHL =3 , f(2) =a $\Rightarrow$ a =3

Ans. 3

LHL =5

RHL =5 , f(2) =k $\Rightarrow$ k =5

Ans. 4

LHL = 5a -2b = f(1) = 11

RHL = 3a +b = f(1) = 11

Solving them we get a=2, b=5

Ans. 5

LHL = 5 , RHL = 2a+b, f(2) =5

LHL = 10 a+b , RHL = 21 , f(10)=21

2a+b =5 and 10a+b =21 solving a=2,b=1

Ans. 6

LHL = πk+1

RHL = -1, f(π)= πk+1 k =-2/π

Ans. 7

LHL = 2k

RHL = 1, f(0)=2k $\Rightarrow$ k =1/2

Ans. 8

LHL =0 , RHL =1,f(0)=0 no value of λ for which it is continuous.

Ans. 9

LHL = 3a+1, RHL = 3b+3 , f(3)= 3a+1

3a+1=3b+3 $\Rightarrow$ 3a-3b=2

Ans. 10

LHL =2a+b+4,RHL = 8,f(2)=8

LHL =14, RHL = 8a+5b,f(4) =14

Thus 2a+b=4, 8a+5b=14 solving we get b=-2,a=3

Differentiation

Ans. 1

Putting U = $x^{\cos x}$ and V = $(\cos x)^{\sin x}$

Finding $\frac{dU}{dx} = x^{\cos x}\left(\frac{\cos x}{x} - \sin x \log x\right)$

Finding $\frac{dV}{dx} = (\cos\ x)^{\sin x}\left(\cos x \log \cos x - \sin x \tan x\right)$

Writing the value of $\frac{dy}{dx}$

Ans. 2

$y = x^x + (\cos x)^{\sin x}$

Let $U = x, V = (\cos x)^{\sin x}$

Y=U+V, so $\frac{dy}{dx} = \frac{dU}{dx} + \frac{dV}{dx}$

Now, $U = x^x \Rightarrow \log U = x \log x$

$$\frac{1}{U}\frac{dU}{dx} = x.\frac{1}{x} + \log x$$

$$\frac{dU}{dx} = x^x(1 + \log x)$$

Similarly $\frac{dV}{dx} = (\cos x)^{\sin x}(\cos x \log \cos x - \sin x \tan x)$

$$\frac{dy}{dx} = x^x(1 + \log x) + (\cos x)^{\sin x}(\cos x \log \cos x - \sin x \tan x)$$

Ans. 3

$$\frac{dx}{dt} = ap\ \cos pt$$
$$\frac{dy}{dt} = -bp \sin\ pt$$
$$\frac{dy}{dx} = -\frac{b}{a} \tan\ pt$$
$$\frac{d^2y}{dx^2} = -\frac{b}{a^2} \sec^3\ pt$$
$$\frac{d^2y}{dx^2}_{at\ t=0} = -\frac{b}{a^2}$$

Ans.4

$$\text{Then, } \frac{dx}{dt} = a\cdot\left[\frac{d}{dt}(\cos t) + \frac{d}{dt}\left(\log\tan\frac{t}{2}\right)\right]$$

$$= a\left[-\sin t + \frac{1}{\tan\frac{t}{2}}\cdot\frac{d}{dt}\left(\tan\frac{t}{2}\right)\right]$$

$$= a\left[-\sin t + \cot\frac{t}{2}\cdot\sec^2\frac{t}{2}\cdot\frac{d}{dt}\left(\frac{t}{2}\right)\right]$$

$$= a\left[-\sin t + \frac{\cos\frac{t}{2}}{\sin\frac{t}{2}}\times\frac{1}{\cos^2\frac{t}{2}}\times\frac{1}{2}\right]$$

$$= a\left[-\sin t + \frac{1}{2\sin\frac{t}{2}\cos\frac{t}{2}}\right]$$

$$= a\left(-\sin t + \frac{1}{\sin t}\right)$$

$$= a\left(\frac{-\sin^2 t + 1}{\sin t}\right)$$

$$= a\frac{\cos^2 t}{\sin t}$$

$$\frac{dy}{dt} = a\frac{d}{dt}(\sin t) = a\cos t$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{a\cos t}{\left(a\frac{\cos^2 t}{\sin t}\right)} = \frac{\sin t}{\cos t} = \tan t$$

Ans. 5

$X = 4\sin^3 t$ & $y = 4\cos^3 t$

$\frac{dx}{dt} = 12\sin^2 t\cos t$ & $\frac{dy}{dt} = -12\cos^2 t\sin t$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = -\cot t$$

$$\frac{d^2y}{dx^2} = \frac{d}{dt}(-\cot t)\frac{dt}{dx} = \frac{\operatorname{cosec}^4 t}{12\cos t}$$

$$\left(\frac{d^2y}{dx^2}\right)_{at\, t=\pi/3} = 8/27$$

Ans. 6

$$Let,\, y=\sqrt{\frac{(x-3)(x^2+4)}{3x^2+4x+5}}$$

Taking logarithm on both side, we have

$$\log y=\frac{1}{2}\left\{\log(x-3)+\log(x^2+4)-\log(3x^2+4x+5)\right\}$$

Differentiating b/s w.r.t. x

$$\frac{1}{y}\frac{dy}{dx}=\frac{1}{2}\left\{\frac{1}{x-3}+\frac{2x}{x^2+4}-\frac{6x+4}{3x^2+4x+5}\right\}$$

$$\therefore \frac{dy}{dx}=\frac{1}{2}\sqrt{\frac{(x-3)(x^2+4)}{3x^2+4x+5}}\left\{\frac{1}{x-3}+\frac{2x}{x^2+4}-\frac{6x+4}{3x^2+4x+5}\right\}$$

Ans.7

Let y = $(\sin x)^x + \tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right)$

$$=e^{\log(\sin x)^x}+\tan^{-1}\frac{1-\tan x}{1+\tan x}$$

$$= e^{x\log(\sin x)}+\tan^{-1}\{\tan(\frac{\pi}{4}-x)\}$$

$$=e^{x\log(\sin x)}+(\frac{\pi}{4}-x)\}$$

$$\Rightarrow \frac{dy}{dx}=e^{x\log(\sin x)}(x\cot x+\log\sin x)-1$$

Ans.8

Taking log both side

Ylogx = x-y differentiating w.r.t x

$$\frac{y}{x}+y^{/}\log x=1-y^{/}$$

$$y^{/}(1+\log x)=\frac{x-y}{x}=\frac{y\log x}{x}$$

$$y^{/}(1+\log x)=\frac{x\log x}{x(1+\log x)}=\frac{\log x}{1+\log x}$$

$$\frac{dy}{dx}=\frac{logx}{(1+lo[gx)]^2}$$

Ans. 9

Put u=x^y and v=y^x and take log both side we get log u = y log x and log v=xlog y

$$\frac{du}{dx}=\frac{y}{x}+y^{/}\log x,\frac{dv}{dx}=\frac{x}{y}y^{/}+\log y$$

$$\frac{du}{dx}+\frac{dv}{dx}=\frac{y}{x}+y^{/}\log x+\frac{x}{y}y^{/}+\log y=0$$

Solving we get dy/dx

Ans. 10

Taking log both side ylog cosx = x log cos y Differentiating wrt x we get

y log cosx = x log cos y

$$y^{/}\log\cos x-y\tan x=\log\cos y-x\tan yy^{/}$$

$$y^{/}(\log\cos x+x\tan y)=\log\cos y+y\tan x$$

$$\frac{dy}{dx}=\frac{\log\cos y+y\tan x}{(\log\cos x+x\tan y)}$$

Maxima minima

Ans.1
Let r and h be the radius and the height of the solid

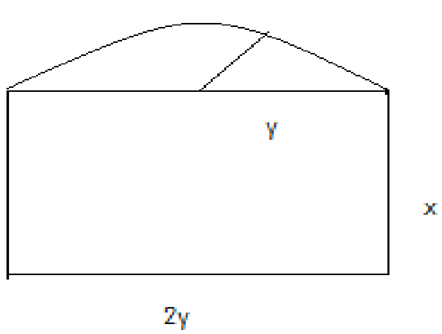
$$V=\frac{1}{2}\pi r^2h,...S=\pi rh+\pi r^2+2rh$$

$$S=\left(\pi+2\right)r.2\frac{V}{\pi r^2}+\pi r^2=\frac{2V\left(\pi+2\right)}{\pi r}+\pi r^2$$

$$\frac{dS}{dr}=0\Rightarrow\frac{2V\left(\pi+2\right)}{\pi r^2}=2\pi r....\Rightarrow\frac{h}{2r}=\frac{\pi}{\pi+2}$$

$$\frac{d^2S}{dr^2}isnegative$$

Ans. 2



For drawing the diagram

2x + 2y + π y = P $\Rightarrow$ x = $\frac{P-2y-\pi y}{2}$

Area = py -2y^2- $\frac{\pi y^2}{2}$

$$\frac{dA}{dy} = 0 \Rightarrow y = \frac{P}{\pi + 4}$$

$$\frac{d^2A}{dy^2} = -4 - \pi < 0 \quad for \;\; y = \frac{P}{\pi + 4}$$

$\therefore$ radius of the semi-circle is $\frac{p}{\pi + 4}$ cm

Ans.3as above

Ans.4

l = b = x, height = y

x^2 + 4xy = a^2 $\Rightarrow \; y = \frac{a^2 - x^2}{4x}$

Volume = $\frac{1}{4}\left(a^2 x - x^3\right)$

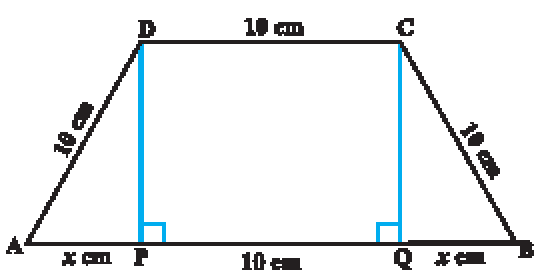
$$\frac{dV}{dx} = 0 \Rightarrow x = \frac{a}{\sqrt{3}}$$

$$\frac{d^2V}{dx^2} = \frac{1}{4}(-6x) \quad \left.\frac{d^2V}{dx^2}\right]_{x=\frac{a}{\sqrt{3}}} < 0$$

Maximum Volume = $\frac{a^3}{6\sqrt{3}}$

Ans 5

The required trapezium is as given in fig. Draw perpendiculars DP and CQ on AB.



Let AP = *x* cm. Note that □APD ~ □BQC. Therefore, QB =*x* cm. Also, by

Pythagoras theorem, DP = QC = $\sqrt{100 - x^2}$.

Let A be the area of the trapezium. Then

$$A \equiv A(x) = \frac{1}{2}(\text{sum of parallel sides})(\text{height})$$
$$= \frac{1}{2}(2x+10+10)\left(\sqrt{100-x^2}\right)$$
$$= (x+10)\left(\sqrt{100-x^2}\right)$$
$$A'(x) = (x+10)\frac{(-2x)}{2\sqrt{100-x^2}} + \left(\sqrt{100-x^2}\right)$$
$$= \frac{-2x^2-10x+100}{\sqrt{100-x^2}}$$

Now $A'(x) = 0$ gives $2x^2 + 10x - 100 = 0$, i.e., $x = 5$ and $x = -10$.
Since x represents distance, it can not be negative.
So, $x = 5$. Now

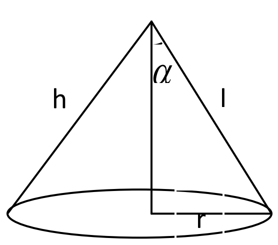
$$A''(x) = \frac{\sqrt{100-x^2}(-4x-10)-(-2x^2-10x+100)\frac{(-2x)}{2\sqrt{100-x^2}}}{100-x^2}$$
$$= \frac{2x^3-300x-1000}{(100-x^2)^{\frac{3}{2}}} \text{ (on simplification)}$$
$$A''(5) = \frac{2(5)^3-300(5)-1000}{(100-(5)^2)^{\frac{3}{2}}} = \frac{-2250}{75\sqrt{75}} = \frac{-30}{\sqrt{75}} < 0$$

Thus, area of trapezium is maximum at $x = 5$ and the area is given by

$$A(5) = (5+10)\sqrt{100-(5)^2} = 15\sqrt{75} = 75\sqrt{3}\text{ cm}^2$$

Ans.6

Let r, h, l ,S and V be the radius, hight, slant hight. surface area and the volume of the cone.



$$S = \pi rl + \pi r^2$$
$$l = \frac{S - \pi r^2}{\pi r}$$
$$and\ V = \frac{1}{3}\pi r^2 h \Rightarrow V^2 = \frac{1}{9}\pi^2 r^4 h^2$$

$$\frac{dv^2}{dr} = 0$$
$$\frac{1}{9}(2rS^2 - 8S\pi r^3) = 0$$
$$\min \Rightarrow \frac{r}{l} = \frac{1}{3}$$
$$now\ \frac{d^2V^2}{dr^2}(at\ S = 4\pi r^2) < o$$
$$\therefore V^2\ is\ \max imum.$$
$$\therefore V\ is\ \max imum.$$

Now
$$\sin\alpha = \frac{r}{l} = \frac{1}{3}$$
$$\alpha = \sin^{-1}(\frac{1}{3}).$$

Ans. 7

Let OC = r be the radius of the cone and OA = h be its height. Let a cylinder with radius OE = x inscribed in the given cone (Fig 6.20). The height QE of the cylinder is given by

$$\frac{QE}{OA} = \frac{EC}{OC} \quad \text{(since } \Delta QEC \sim \Delta AOC)$$

or $$\frac{QE}{h} = \frac{r - x}{r}$$

or $$QE = \frac{h(r - x)}{r}$$

Let S be the curved surface area of the given cylinder. Then

$$S \equiv S(x) = \frac{2\pi x\, h(r - x)}{r} = \frac{2\pi h}{r}(rx - x^2)$$

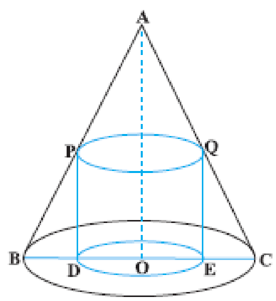


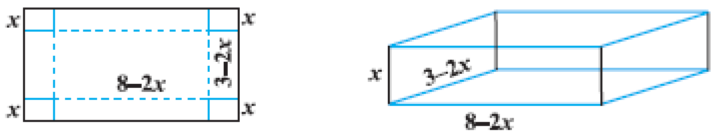
Fig 6.20

or $$\begin{cases} S'(x) = \dfrac{2\pi h}{r}(r - 2x) \\ S''(x) = \dfrac{-4\pi h}{r} \end{cases}$$

Now $S'(x) = 0$ gives $x = \frac{r}{2}$. Since $S''(x) < 0$ for all x, $S''\left(\frac{r}{2}\right) < 0$. So $x = \frac{r}{2}$ is a point of maxima of S. Hence, the radius of the cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

Ans.8

Let x metre be the length of a side of the removed squares. Then, the height of the box is x, length is $8 - 2x$ and breadth is $3 - 2x$ (Fig 6.25). If $V(x)$ is the volume of the box, then



(a) (b)

Fig 6.25

$$V(x) = x(3 - 2x)(8 - 2x)$$
$$= 4x^3 - 22x^2 + 24x$$

Therefore $\begin{cases} V'(x) = 12x^2 - 44x + 24 = 4(x-3)(3x-2) \\ V''(x) = 24x - 44 \end{cases}$

Now $V'(x) = 0$ gives $x = 3, \frac{2}{3}$. But $x \neq 3$ (Why?)

Thus, we have $x = \frac{2}{3}$. Now $V''\left(\frac{2}{3}\right) = 24\left(\frac{2}{3}\right) - 44 = -28 < 0$.

Therefore, $x = \frac{2}{3}$ is the point of maxima, i.e., if we remove a square of side $\frac{2}{3}$ metre from each corner of the sheet and make a box from the remaining sheet, then the volume of the box such obtained will be the largest and it is given by

$$V\left(\frac{2}{3}\right) = 4\left(\frac{2}{3}\right)^3 - 22\left(\frac{2}{3}\right)^2 + 24\left(\frac{2}{3}\right)$$
$$= \frac{200}{27} \text{ m}^3$$

Integral

Ans.1

Multiplying sin(a – b) in N^r & D^r. Apply In N^r expressing sin(a – b) = sin [(x-b)-(x-a)]
Applying the formulae sin(A-B)

Simplification

$$I = \frac{1}{\sin(a-b)} \int \frac{\sin(x-b)\cos(x-a) - \cos(x-b)\sin(x-a)}{\cos(x-a)\cos(x-b)} dx$$
$$I = \frac{1}{\sin(a-b)} \int (\tan(x-b) - \tan(x-a)) dx$$

Getting the result $I = \frac{1}{\sin(a-b)}\left[\log\left|\frac{\cos(x-a)}{\cos(x-b)}\right|\right] + C$

Ans.2

$$\int e^x \frac{x^2+1}{(x+1)^2}dx = \int e^x\left[\frac{x-1}{x+1}+\frac{2}{(x+1)^2}\right]dx$$

$$= \int e^x \frac{x-1}{x+1}dx + \int e^x \frac{2}{(x+1)^2}dx$$

$$= \frac{x-1}{x+1}e^x - \int\left[\frac{d}{dx}\frac{x-1}{x+1}\int e^x dx\right]dx + \int e^x \frac{2}{(x+1)^2}dx$$

$$= \frac{x-1}{x+1}e^x - \int e^x \frac{2}{(x+1)^2}dx + \int e^x \frac{2}{(x+1)^2}dx + C$$

$$= \frac{x-1}{x+1}e^x + C$$

Ans.3

$$I = \int \frac{x+1}{(x+3)^3}e^x\,dx = \int \frac{x+3-2}{(x+3)^3}e^x dx$$

$$= \int\left(\frac{1}{(x+3)^2} - \frac{2}{(x+3)^3}\right)e^x dx$$

$$= \int e^x\left(f(x) + f'(x)\right)dx \text{ where } f(x) = \frac{1}{(x+3)^2}$$

$$= e^x f(x) + c = e^x \frac{1}{(x+3)^2} + c$$

Ans.4

$$I = \int \frac{\cos x dx}{(1+\sin x)(2+\sin x)}; \textit{put} \sin x = t \Rightarrow \cos x dx = dt$$

$$I = \int \frac{dy}{(1+t)(2+t)} = \int\left(\frac{1}{1+t} - \frac{1}{2+t}\right)dt = \int \frac{dt}{1+t} - \int \frac{dt}{2+t}$$

$$I = \log\left|\frac{1+t}{2+t}\right| + c$$

Putting t = sin x we obtain $I = \log\left|\frac{1+\sin x}{2+\sin x}\right| + c$

Ans.5

$$I=\int e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx=\int e^x\left(\frac{1-\sin x}{2\sin^2\frac{x}{2}}\right)dx$$

$$I==\int e^x\left(\frac{1}{2\sin^2\frac{x}{2}}-\frac{2\sin(x/2)\cos(x/2)}{2\sin^2\frac{x}{2}}\right)$$

$$I=\int e^x\left(\frac{1}{2}\cos ec^2(x/2)-\cot^2(x/2)\right)$$

I = $e^x\cot(x/2)+c$ Since $\int e^x\left(f'(x)+f(x)\right)dx=e^x f(x)+c$

Ans 6

$$I=\frac{1}{2}\int\frac{dx}{\frac{1}{2}\sin x+\frac{\sqrt{3}}{2}\cos x}=\frac{1}{2}\int\sec\left(x-\frac{\pi}{6}\right)dx$$

$$I=\frac{1}{2}\log\left|\sec\left(x-\frac{\pi}{6}\right)+\tan\left(x-\frac{\pi}{6}\right)\right|+C$$

Ans .7 $Put....(\sin x-\cos x)=t\Rightarrow$(cosx+sinx)dx=dt and 1-sin2x=t^2 $\Rightarrow$sin2x =1-t^2 putting inI we get

$$I=\int\frac{dt}{25-16t^2}=\frac{1}{40}\log\left|\frac{5+4(\sin x-\cos x)}{5-4(\sin x-\cos x)}\right|+c$$

Ans.8

$$\text{I}=\int\frac{dx}{x(x^n+1)}$$

Substitute x^n = t $\Rightarrow$ dx = $\frac{1}{n\,x^{n-1}}dt$

$$\Rightarrow \text{I}=\frac{1}{n}\int\frac{dt}{t(t+1)}$$

$$=\frac{1}{n}\int\left(\frac{1}{t}-\frac{1}{t+1}\right)dt$$

$$=\frac{1}{n}\log\left|\frac{t}{t+1}\right|+c$$

$$=\frac{1}{n}\log\left|\frac{x^n}{x^n+1}\right|+c$$

Ans 9

$$Let\ f(x)=[|x-1|+|x-2|+|x-3|],$$

$$f(x)=[|x-1|+|x-2|+|x-3|]=\begin{cases}4-x, & if\ 1<x<2\\ x, & if\ 2<x<3\\ 3x-6, & if\ 3<x<4\end{cases}$$

$$We\ know\ that, \int_a^b f(x)dx=\int_a^c f(x)dx+\int_c^d f(x)dx+\int_d^b f(x)dx, \text{ where } a<c<d<b$$

$$\int_1^4 [|x-1|+|x-2|+|x-3|]dx=\int_1^2 [|x-1|+|x-2|+|x-3|]dx+\int_2^3 [|x-1|+|x-2|+|x-3|]dx+\int_3^4 [|x-1|+|x-2|+|x-3|]dx$$

$$=\int_1^2 (4-x)dx+\int_2^3 xdx+\int_3^4 (3x-6)dx$$

$$=\frac{19}{2}$$

Ans.10

$$I=\int_0^{\pi}\frac{x\tan x}{\sec x+\tan x}dx \text{ ------------ (1)}$$

$$I=\int_0^{\pi}\frac{(\pi-x)\tan(\pi-x)}{\sec(\pi-x)+\tan(\pi-x)}dx$$

$$I=\int_0^{\pi}\frac{(\pi-x)\tan x}{\sec x+\tan x}dx \text{ ------------(2)}$$

Adding (1) & (2)

$$2I=\int_0^{\pi}\frac{\pi\tan x}{\sec x+\tan x}dx$$

$$=\pi\int_0^{\pi}\frac{\tan x}{\sec x+\tan x}dx$$

$$=\pi\int_0^{\pi}(\tan x\sec x-\tan^2 x)dx$$

$$=\pi\int_0^{\pi}(\tan x\sec x-\sec^2 x+1)dx$$

$$I = \frac{1}{2}\left[\ \pi\left(\sec x - \tan x + x\right)\ \right]_0^{\pi}$$

$$= \frac{1}{2}\pi(\pi - 2)$$

THREE DIMENSIONAL GEOMETRY

Ans.1

Here $\vec{a}_1 = \hat{i} + 2\hat{j} + 3\hat{k}, \vec{b}_1 = \hat{i} - 3\hat{j} + 2\hat{k}$

$\vec{a}_2 = 4\hat{i} + 5\hat{j} + 6\hat{k}, \vec{b}_2 = 2\hat{i} + 3\hat{j} + \hat{k}$

shortest distance between lines = $\dfrac{(\vec{a}_2 - \vec{a}_1).(b_1 \times b_2)}{\left|\vec{b}_1 \times \vec{b}_2\right|}$

$\vec{a}_2 - \vec{a}_1 = 3\hat{i} + 3\hat{j} + 3\hat{k}, \vec{b}_1 \times \vec{b}_2 = -9\hat{i} + 3\hat{j} + 9\hat{k}$

$\left|\vec{b}_1 \times \vec{b}_2\right| = \sqrt{171}$, putting the values, after cal. We get

shortest distance = $\dfrac{3}{\sqrt{19}}$ units

Ans.2

Here

$\overrightarrow{a_1} = \hat{i} + \hat{j}, \quad \overrightarrow{b_1} = 2\hat{i} - \hat{j} + \hat{k},$

$\overrightarrow{a_2} = 2\hat{i} + \hat{j} - \hat{k}, \quad \overrightarrow{b_2} = 3\hat{i} - 5\hat{j} + 2\hat{k}$

$\therefore \overrightarrow{a_2} - \overrightarrow{a_1} = \hat{i} - \hat{k}, \vec{b}1 \times \overrightarrow{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix}$

$= 3\hat{i} - \hat{j} - 7\hat{k}$

$\left|\vec{b}_1 \times \overrightarrow{b_2}\right| = \sqrt{59}$

$\therefore$ shortest distance = $\left|\dfrac{(\vec{b_1}\times\vec{b_2}).(\vec{a_2}-\vec{a_2})}{|\vec{b_1}\times\vec{b_2}|}\right| = \dfrac{10}{\sqrt{59}}$

Ans.3

Equation of the plane

$$\begin{vmatrix} x-3 & y+1 & z-2 \\ 2 & 3 & 2 \\ -4 & 0 & 4 \end{vmatrix} = 0$$

$i.e., 3x-4y+3z-19=0$

Now the perp. Distance from (6,5,9)to this plane is

$$= \frac{|3.6-4.5+3.9-19|}{\sqrt{9+16+9}}$$

$$= \frac{6}{\sqrt{34}} units$$

Ans.4

Equation of line AB is $\dfrac{x-1}{1}=\dfrac{y-3}{-1}=\dfrac{z-4}{1}$

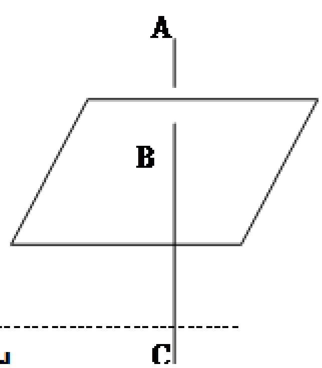
Co-ordinates of B (λ+1,-λ+3, λ+4)

t lies on the plane then λ+1+λ-3 +λ +4 =5

⇨ λ = 1

So, Co-ordinate of B is (2,2,5)

Therefore image of C is (3,5,6)



Let no. of operated machine = x & manually operated

Ans.5. Let Co-Ord. of A′ be (∝, β,γ), the image of A (1, 2, - 1) and R be the mid-point of AA'

$\therefore$ Coor.of $R = \left(\dfrac{\alpha+1}{2}, \dfrac{\beta+2}{2}, \dfrac{\gamma-1}{2}\right)$

Equn.of AA′ is $\dfrac{x-1}{2} = \dfrac{y-2}{1} = \dfrac{z+1}{-1}$

Co.ord. of R on the line in terms of λ as general pt. (2λ+1, λ+2, -λ-1)

This pt. lies on the plane

⇒2(2λ+1)+(λ+2)-(-λ-1)=2

⇒6λ=-3

⇒λ= - ½

$$\therefore \frac{\alpha+1}{2} = \frac{-1}{2}.2+1 \Rightarrow \alpha = -1$$

$$\therefore \frac{\beta+2}{2} = -\frac{1}{2}+2 \Rightarrow \beta = 1$$

$$\frac{\gamma-1}{2} = -\left(\frac{-1}{2}\right)-1 \Rightarrow \gamma = 0$$

∴The coord.of image is (-1, 1, 0)

Ans 6

$$Given,\ \vec{r} = \hat{i} - \hat{j} + \lambda\left(2\hat{i} + \hat{k}\right);\ \vec{r} = 2\hat{i} - \hat{j} + \mu\left(\hat{i} - \hat{j} - \hat{k}\right)$$

$$S.D. = \left|\frac{(\vec{a}_2 - \vec{a}_1)\cdot\left(\vec{b}_1 \times \vec{b}_2\right)}{\left|\vec{b}_1 \times \vec{b}_2\right|}\right| = \frac{1}{14} unit \neq 0$$

hence the given line doesn't intersect.

Ans. 7

The equations of the given planes are

$$\vec{r}\cdot\left(\hat{i}+2\hat{j}+3\hat{k}\right)-4=0 \qquad ...(1)$$

$$\vec{r}\cdot\left(2\hat{i}+\hat{j}-\hat{k}\right)+5=0 \qquad ...(2)$$

The equation of the plane passing through the line intersection of the plane given in equation (1) and equation (2) is

$$\left[\vec{r}\cdot\left(\hat{i}+2\hat{j}+3\hat{k}\right)-4\right]+\lambda\left[\vec{r}\cdot\left(2\hat{i}+\hat{j}-\hat{k}\right)+5\right]=0$$

$$\vec{r}\cdot\left[(2\lambda+1)\hat{i}+(\lambda+2)\hat{j}+(3-\lambda)\hat{k}\right]+(5\lambda-4)=0 \qquad ...(3)$$

The plane in equation (3) is is passing through (1,0,-2)

2λ+1-6+2λ+5λ- 4=0 => 9λ=9 =>λ=1

Substituting $\lambda = \frac{7}{19}$ in equation (3), we obtain r.(3i+3j+2k+1)=0 or 3x+3y+2z+1=0

Ans.8

Writing in cartesian form
Eq^n of the plane passing through the line of intersection of
X + 3y – z =0 & y + 2z =0 is
X + 3y – z + λ (y + 2z)=0
Passing through (2, 1, -1)
$\Rightarrow \lambda = 6$

Eq^n is x + 9y +11z = 0

Ans.9

Given line is $\frac{x+1}{2}=\frac{3y+5}{9}=\frac{3-z}{-6}$.

$$or, \frac{x-(-1)}{2}=\frac{y-\left(\frac{5}{3}\right)}{3}=\frac{z-3}{6}.$$

Drs of the line is <2,3,6> Given equation of plane: $10x+2y-11z=3$
Drs of normal to the plane is <10,2,-11>
Let θ be the angle between the line and plane,then

$$\sin\theta=\frac{\vec{b}\cdot\vec{n}}{|\vec{b}|\;|\vec{n}|}=\frac{(2\hat{i}+3\hat{j}+6\hat{k}).\,(10\hat{i}+2\hat{j}-11\hat{k})}{|2\hat{i}+3\hat{j}+6\hat{k}|\;|10\hat{i}+2\hat{j}-11\hat{k}|}$$

$$=-\frac{8}{21}$$

$$\therefore \theta=\sin^{-1}\left(-\frac{8}{21}\right)$$

Ans.10

eq. of plane passing through one given point and perpendicular to planes

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$$

$$\Rightarrow \begin{vmatrix} x+1 & y+1 & z-2 \\ 1 & 2 & -3 \\ 5 & -4 & 3 \end{vmatrix}=0$$

$\Rightarrow$ 3x + 9y + 7z – 2 = 0 , after simplification

Probability

Ans.1

$\vec{a} \; X \; \vec{b} = \;\; 32\,\hat{\imath} - \; \hat{\jmath} \; - 14\,\hat{k}$

$\vec{d} = \lambda(\vec{a} \; X \; \vec{b})$

$\vec{d} = \frac{5}{3}(32\,\hat{\imath} - \; \hat{\jmath} \; - 14\,\hat{k})$

Ans.2

. Consider $\left|\hat{a}+\hat{b}\right|^2 = 1+1+2\cos\theta$

$=2(1+\cos\theta)$

$=4\cos^2\frac{\theta}{2}$

$\Rightarrow \cos\frac{\theta}{2} = \frac{1}{2}\left|\hat{a}+\hat{b}\right|$

Ans.3

$|\vec{a}| = |\vec{b}| = |\vec{c}| = \lambda; \vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$

$\left\|\vec{a}+\vec{b}+\vec{c}\right\|^2 = \left(\vec{a}+\vec{b}+\vec{c}\right).\left(\vec{a}+\vec{b}+\vec{c}\right) = 3\lambda^2$.

$\left|\vec{a}+\vec{b}+\vec{c}\right| = \sqrt{3}\lambda$

$|\vec{a}| = |\vec{b}| = |\vec{c}| = \lambda; \vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$

$\left\|\vec{a}+\vec{b}+\vec{c}\right\|^2 = \left(\vec{a}+\vec{b}+\vec{c}\right).\left(\vec{a}+\vec{b}+\vec{c}\right) = 3\lambda^2$

$\left|\vec{a}+\vec{b}+\vec{c}\right| = \sqrt{3}\lambda$

Let θ be the angle between $(\vec{a}+\vec{b}+\vec{c}) and \vec{a}$ then $\cos\theta = \frac{\vec{a}.(\vec{a}+\vec{b}+\vec{c})}{|\vec{a}|\left|\vec{a}+\vec{b}+\vec{c}\right|} = \frac{1}{\sqrt{3}}$

Similarly we can prove that($\vec{a}+\vec{b}+\vec{c}$) makes the same angle with the other two vectors

Ans.4

$Let\ \hat{c} = \hat{a} + \hat{b}$

$|\hat{c}| = \sqrt{|\hat{a}|^2 + |\hat{b}|^2 + 2|\hat{a}||\hat{b}|\cos\theta}$

$Since\ \hat{a}, \hat{b}\ \&\ \hat{c}\ are\ unit\ vectors.$

$2|\hat{a}||\hat{b}|\cos\theta = -1$

$Now, let\ \hat{d} = \hat{a} - \hat{b}$

$|\hat{d}| = \sqrt{|\hat{a}|^2 + |\hat{b}|^2 - 2|\hat{a}||\hat{b}|\cos\theta}$

$|\hat{d}| = \sqrt{3}$

Ans.5

We have $[\vec{a} + \vec{b} \quad \vec{b} + \vec{c} \quad \vec{c} + \vec{a}] = [(\vec{a} + \vec{b}) X (\vec{b} + \vec{c})].(\vec{c} + \vec{a})$

=(axb + axc+ bxb + bxc).(c+a) = (axb +axc + bxc). (c + a) (as bxb = 0)

= (axb).c + (bxc).a =2 [abc] (expanding and using (axb).a = 0 etc.

Ans.6

We have $\left(\vec{a} + \vec{b} + \vec{c}\right)^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2\vec{a}\vec{b} + 2\vec{b}\vec{c} + 2\vec{c}\vec{a} = 1^2 + 4^2 + 2^2 + 2(\vec{a}\vec{b} + \vec{b}\vec{c} + \vec{c}\vec{a})$ Since $\left(\vec{a} + \vec{b} + \vec{c}\right)$=0,

we have $0 = 21 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) \Rightarrow \vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = \frac{-21}{2} \Rightarrow .$

Ans.7

$\vec{\alpha} = 3\hat{i} - \hat{j}$

$\vec{\beta} = 2\hat{i} + \hat{j} - 3\hat{k}.$

$\vec{\beta}_1$ is parallel to $\vec{\alpha} \Rightarrow \vec{\beta}_1 = k\,\vec{\alpha}$

$\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2 \Rightarrow \vec{\beta}_2 = \vec{\beta} - k\,\vec{\alpha}$

$\vec{\beta}_2 \perp \vec{\alpha}$

$\Rightarrow \vec{\beta}_2 \cdot \vec{\alpha} = 0 \Rightarrow k = 1/2$

$\vec{\beta}_1 = \frac{1}{2}(3\hat{i} - \hat{j})$

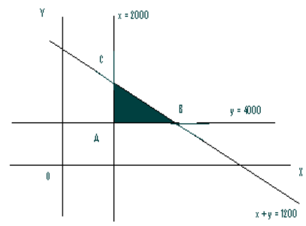
$\vec{\beta}_2 = \frac{1}{2}(\hat{i} + 3\hat{j} - 6\hat{k}.)$

LPP

Ans.1

Let she invests Rs. x in saving certificates and Rs. y in National saving bonds

Then LPP is



To maximize Z=0.08x+0.1y

Subject to constraints

$x \geq 2000, y \geq 4000, x + y \leq 12000$

corner points of feasible region ABC are A(2000,4000), B(8000,4000),c(2000,10000)

at A, Z=160+400=560

at B, Z=640+400=1040

at C,Z=160+1000=1160

thusRs. 2000 should be invested in saving certificates and Rs.10000 in National saving bonds. Maximum yearly income is Rs. 1160

Ans.2

Let x be the no. of fans and y be the no. of sewing machines

If p be the total profit ,

P= 22x+18y

$360x+240y \leq 5760$. For correct graph, ---------- (1)

i.e., $3x+2y \leq 48$.

$X+y \leq 20$.

$x \geq 0, y \geq 0$ ------------------------------- (2)

points	value of p
(0, 0)	0
(16, 0)	352
(8,12)	392
(0, 20)	360

the dealer gets a maximum profit of Rs. 392 when he purchase and sells 8 fans and 12 sewing machines. Investment in fans =360.×8=2880.

Investment in sewing machines =240×12= 2880. --------------------(3)

Ans.3

Let the diet contain x units of food F_1 and y units of food F_2. Therefore,

$x \geq 0$ and $y \geq 0$, The given information can be complied in a table as follows.

	Vitamin A (units)	**Mineral (units)**	**Cost per unit (Rs)**
Food F_1 (x)	**3**	**4**	**4**
Foo F_2 (y)	**6**	**3**	**6**
Requirement	**80**	**100**	

The cost of food F_1 is Rs 4 per unit and of Food F_2 is Rs 6 per unit. Therefore, the constraints are

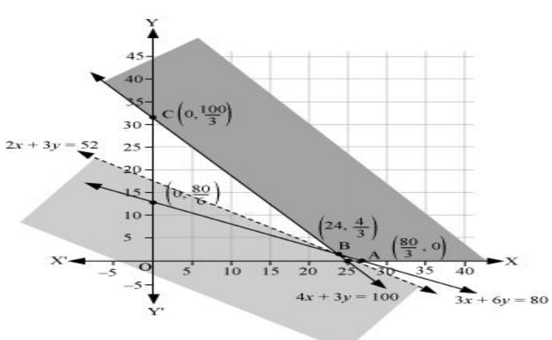
$3x + 6y \geq 80$; $4x + 3y \geq 100$; $x, y \geq 0$

Total cost of the diet, $Z = 4x + 6y$

The mathematical formulation of the given problem is

Minimise $Z = 4x + 6y$ … (1)subject to the constraints,$3x+ 6y \geq 80$ … (2),$4x + 3y \geq 100$ … (3)

$x, y \geq 0$ … (4)



Corner point	Z = 4x + 6y	
$A\left(\frac{80}{3}, 0\right)$	$\frac{320}{3} = 106.67$	
$B\left(24, \frac{4}{3}\right)$	104	→ Minimum
$C\left(0, \frac{100}{3}\right)$	200	

As the feasible region is unbounded, therefore, 104 may or may not be the minimum value of Z.For this, we draw a graph of the inequality, $4x + 6y < 104$ or $2x + 3y < 52$, and check whether the resulting half plane has points in common with the feasible region or not.

It can be seen that the feasible region has no common point with $2x + 3y < 52$

Therefore, the minimum cost of the mixture will be Rs 104.

Ans.4

Let the cottage industry manufacture x pedestal lamps and y wooden shades. Therefore,

$x \geq 0$ and $y \geq 0$

The given information can be compiled in a table as follows.

	Lamps	Shades	Availability
Grinding/Cutting Machine (h)	2	1	12
Sprayer (h)	3	2	20

The profit on a lamp is Rs 5 and on the shades is Rs 3. Therefore, the constraints are

$2x + y \leq 12$

$3x + 2y \leq 20$

Total profit, Z = $5x + 3y$

The mathematical formulation of the given problem is

Maximize Z = $5x + 3y$ … (1)

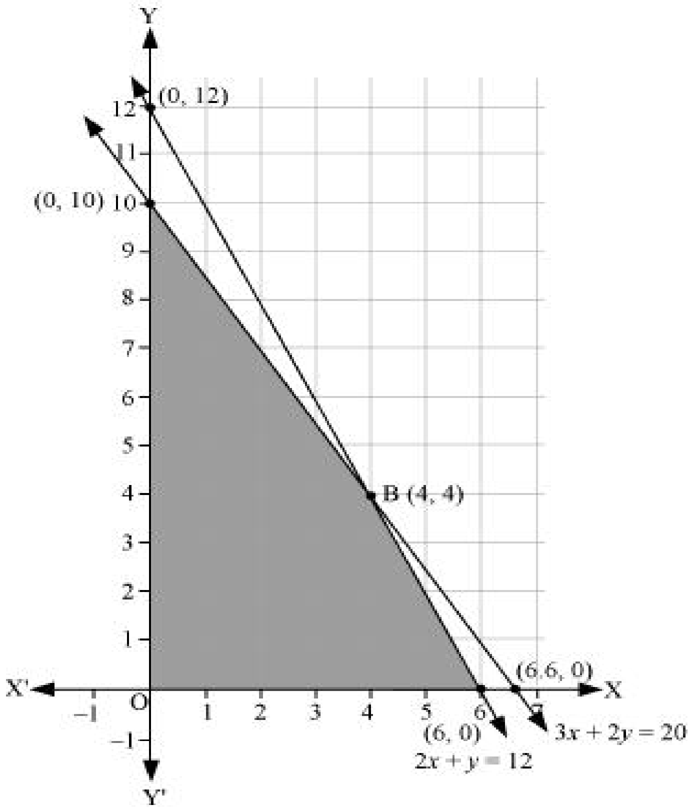
subject to the constraints,

$2x + y \leq 12$ … (2)

$3x + 2y \leq 20$ … (3)

$x, y \geq 0$ … (4)

The feasible region determined by the system of constraints is as follows.



The corner points are A (6, 0), B (4, 4), and C (0, 10).

The values of Z at these corner points are as follows

Corner point	**Z = 5*x* + 3*y***	
A(6, 0)	30	
B(4, 4)	32	→ Maximum
C(0, 10)	30	

The maximum value of Z is 32 at (4, 4).

Thus, the manufacturer should produce 4 pedestal lamps and 4 wooden shades to maximize his profits

Ans. 5

Let the airline sell x tickets of executive class and y tickets of economy class.

The mathematical formulation of the given problem is as follows.

Maximize $z = 1000x + 600y$ … (1)

subject to the constraints,

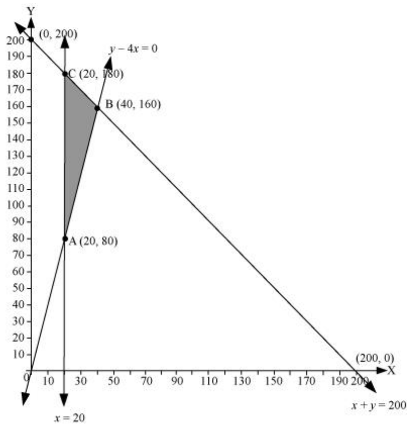
$x + y \le 200$ ...(2)

$x \ge 20$ ...(3)

$y - 4x \ge 0$ ...(4)

$x, y \ge 0$ ...(5)

The feasible region determined by the constraints is as follows.



The corner points of the feasible region are A (20, 80), B (40, 160), and

C (20, 180).

The values of *z* at these corner points are as follows.

Corner point	**z = 1000*x* + 600*y***	
A (20, 80)	68000	
B (40, 160)	136000	→ Maximum
C (20, 180)	128000	

The maximum value of *z* is 136000 at (40, 160).

Thus, 40 tickets of executive class and 160 tickets of economy class should be sold to maximize the profit and the maximum profit is Rs 136000.

Probability

Ans.1

P(A) = 1/4, P(B) = 35/100 & P(C) = 40/100

P(E/A) = 5/100, P(E/B) = 4/100, P(E/C) = 2/100

P(C/E) =16/19

Ans.2

Consider A_1 : suffers from TB, $P\left(A_1\right) = 0.001)$

A_2 : *person donot suffer fromTB*, $P\left(A_2\right) = 0.999$

C: Doctor diagnoses correctly

Then $P(C/A_1) = 0.99$ *and* $P(C/A_2) = 0.001$

By Baye 's theorem $P(A_1/C) = \dfrac{P(A_1).P(C/A_1)}{P(A_1).P(C/A_1) + P(A_2).P(C/A_2)} = \dfrac{110}{221}$

Ans.3

. E_1: Heads on both faces coin

E_2 : biased coin

E_3 : Unbiased coin

$P(E_1) = P(E_2) = P(E_3) = 1/3$

A = Getting a head.

$P(A/E_1) = 1$, $P(A/E_2) = 3/4$, $P(A/E_3) = 1/2$.

$$P(E_1/A) = \frac{P(Ei)P(A/Ei)}{\sum P(Ei)P(A/Ei)}, i = 1,2,3$$

= --- ---

= 5/9 Ans.

Ans.4

Let E_1 and E_2 be the respective events that the student knows the answer and he guesses the answer.

Let A be the event that the answer is correct.

$$\therefore P\left(E_1\right) = \frac{3}{4}$$

$$P\left(E_2\right) = \frac{1}{4}$$

The probability that the student answered correctly, given that he knows the answer, is 1.

$\therefore$ P $(A|E_1) = 1$

Probability that the student answered correctly, given that he guessed, is $\frac{1}{4}$.

$\therefore P(A|E_2) = \frac{1}{4}$

The probability that the student knows the answer, given that he answered it correctly, is given by $P(E_1|A)$.

By using Bayes' theorem, we obtain

$$P(E_1|A) = \frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A|E_2)}$$

$$= \frac{\frac{3}{4}\cdot 1}{\frac{3}{4}\cdot 1 + \frac{1}{4}\cdot\frac{1}{4}}$$

$$= \frac{\frac{3}{4}}{\frac{3}{4} + \frac{1}{16}}$$

$$= \frac{\frac{3}{4}}{\frac{13}{16}}$$

$$= \frac{12}{13}$$

Ans.5

Let E_1 and E_2 be the respective events of choosing a diamond card and a card which is not diamond.

Let A denote the lost card.

Out of 52 cards, 13 cards are diamond and 39 cards are not diamond.

$$\therefore P(E_1) = \frac{13}{52} = \frac{1}{4}$$

$P(E_2) = \frac{39}{52} = \frac{3}{4}$ Two cards can be drawn out of 12 diamond cards in ${}^{12}C_2$ ways.

Similarly, 2 diamond cards can be drawn out of 51 cards in ${}^{51}C_2$ ways. The probability of getting two cards, when one diamond card is lost, is given by P (A|E_1).

$$P(A|E_1) = \frac{{}^{12}C_2}{{}^{51}C_2} = \frac{12!}{2!\times 10!} \times \frac{2!\times 49!}{51!} = \frac{11\times 12}{50\times 51} = \frac{22}{425}$$

When the lost card is not a diamond, there are 13 diamond cards out of 51 cards.

Two cards can be drawn out of 13 diamond cards in ${}^{13}C_2$ ways whereas 2 cards can be drawn out of 51 cards in ${}^{51}C_2$ ways.

The probability of getting two cards, when one card is lost which is not diamond, is given by P (A|E_2).

$$P(A|E_2) = \frac{{}^{13}C_2}{{}^{51}C_2} = \frac{13!}{2!\times 11!} \times \frac{2!\times 49!}{51!} = \frac{12\times 13}{50\times 51} = \frac{26}{425}$$

The probability that the lost card is diamond is given by P (E_1|A).

$$P(E_1|A) = \frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A|E_2)}$$

$$= \frac{\frac{1}{4}\cdot\frac{22}{425}}{\frac{1}{4}\cdot\frac{22}{425} + \frac{3}{4}\cdot\frac{26}{425}}$$

$$= \frac{\frac{1}{425}\left(\frac{22}{4}\right)}{\frac{1}{425}\left(\frac{22}{4} + \frac{26\times 3}{4}\right)}$$

$$= \frac{\frac{11}{2}}{25}$$

$$= \frac{11}{50}$$

Ans.6

Let A be the event that the insured person meets with an accident and E_1, E_2 and

E_3 are the events that the person is a scooter, car and truck driver respectively.

Then we have to find P(E_1/A) .

Total number of insured persons = 2000+4000+6000 = 12000.

P (E_1) =2000/12000 =1/6; P (E_2) = 4000/12000 = 1/3; P (E_3) = 6000/12000 = ½

Also P (A/E_1) = 0.01; P (A/E_2) = 0.03 and P (A/E_3) = 0.15

Hence by Bye's theorem we have

$$P(E_1/A)=\frac{P(A/E_1)P(E_1)}{P(A/E_1)P(E_1)+P(A/E_2)P(E_2)+P(A/E_3)P(E_3)}$$

$$=\frac{0.01\times 1/6}{0.01\times 1/6+0.03\times 1/3+0.15\times 1/2} = 1/52.$$

Ans7.

Let E denote the event that the person selected is actually having HIV and A

the event that the person's HIV test is diagnosed as +ive. We need to find P(E|A).

Also E ☐☐ denotes the event that the perso

We have $P(E) = 0.1\% = \frac{0.1}{100} = 0.001$

$P(E') = 1 - P(E) = 0.999$

$P(A|E)$ = P(Person tested as HIV+ive given that he/she is actually having HIV)

$= 90\% = \frac{90}{100} = 0.9$

$P(A|E')$ = P(Person tested as HIV +ive given that he/she is actually not having HIV)

$= 1\% = \frac{1}{100} = 0.01$

Now, by Bayes' theorem

$$P(E|A) = \frac{P(E)P(A|E)}{P(E)P(A|E)+P(E')P(A|E')}$$

$$= \frac{0.001\times 0.9}{0.001\times 0.9+0.999\times 0.01} = \frac{90}{1089}$$

$= 0.083$ approx.

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